

An Intuitive Understanding About Fourier Transform

Mathematically, Fourier Transform transforms the signal from the time domain to the frequency domain. This is a very precise, whereas counter-intuitive description.

I offer a more intuitive way of understanding Fourier Transform in this write-up.

First, we must dig into a fundamental question: how do we encode information? In a nutshell, **information is encoded through changes**, i.e., the change of voltage, the change of current, the change of brightness, the change of color, the change of smell, or, in general, the change of anything. Since change is an abstract object, we use some medium or carrier to reflect those changes, such as voltage, current, light, color, and smell, from our previous example.

The interesting thing is that we human beings often pay more attention to those mediums or carriers as opposed to the essence - the changes.

Fourier Transform is the tool that focuses purely on the changes while avoiding those carriers.

$$g(t) \Leftrightarrow G(f) = \int g(t) e^{-j2\pi f t} dt, G(f) \text{ is a function of } f, \text{ a.k.a., changes.}$$

For a piece of input information, Fourier Transform is an "accounting" algorithm. It figures out the weight associated with each rate of change, i.e., f . Here, the rate of change f can be low or large. When $f=0$, there is no change (we call it direct current or DC signal).

When f is large, it is a fast change.

Mathematically, we also prove that Fourier Transform does not change the energy or power associated with the input information. Therefore, Fourier Transform is an equivalent transformation of the same piece of information, at least from an energy/power perspective.

It is like for the same object, Fourier Transform gives us a perspective of viewing this object from the most concerning viewpoint, i.e., the information encoding/decoding point.

As a side note, Fourier Transform departs from Fourier Series while removing some unnecessary attachments from Fourier Series to arrive at a more fundamental and impactful tool, similarly, for LU factorization and Gaussian elimination.

For our business, such as writing a paper, or mentoring an individual, among many others, reducing entropy should be the golden direction to pursue. However, the correct way of reducing entropy matters!

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